Formulas for gears calculation – internal gears

Contens

- *Internal spur gears with normal profile*
- *Internal spur gears with corrected profile*
  - *Without center distance variation*
  - *With center distance variation*
- *Internal helical gears with normal profile*
- *Internal helical gears with corrected profile*
  - *Without center distance variation*
  - *With center distance variation*
- *Length of contact and contact radius R_a*
- *Interference*
- *Dimension over pins and balls*

Meaning of symbols			
a	*Center distance*	m	*Module*
α	*Pressure angle*	Q	*Dimension over pins or balls*
β	*Helix angle*	r	*Radius*
d	*Diameter*	R_a	*Radius to start of active profile*
g	*Length of contact*	s	*Tooth thickness on diameter d*
g_1	*Legth of recession*	$\bar{s}_{os}$	*Chordal thickness*
g_2	*Length of approach*	t	*Pitch*
h_f	*Dedendum*	w	*Chordal thickness over z' teeth (spur gears)*
h_k	*Addendum*	W	*Chordal thickness over z' teeth (helical gears)*
h_0	*Corrected addendum*	z	*Number of teeth*
h_r	*Whole depth*	x	*Profile correction factor*
l	*Tooth space*		
Meaning of indices			
b	*Reffered to rolling diameter*	n	*Reffered to normal section*
c	*Referred to roll diameter of basic rack*	o	*Reffered to pitch diameter*
f	*Reffered to root diameter*	q	*Reffered to the diameter throug balls center*
g	*Reffered to base diameter*	r	*Reffered to balls*
k	*Refferred to outside diameter*	s	*Reffered to transverse section*
i	*Refferred to equivalent*	w	*Reffered to tool*

Internal spur gears with normal profile

$$d_o = m \cdot z$$
$$t_o = m \cdot \pi$$
$$d_g = d_o \cdot \cos \alpha_o$$
$$t_g = t_o \cdot \cos \alpha_o$$
$$h_k = m$$
$$h_f = \frac{7}{6} \cdot m \quad \text{or} \quad h_f = \frac{5}{4} \cdot m$$
$$h_r = h_k + h_f$$
$$d_k = d_o - 2h_k$$
$$d_f = d_o + 2h_f$$
$$s_o = \frac{m \cdot \pi}{2}$$

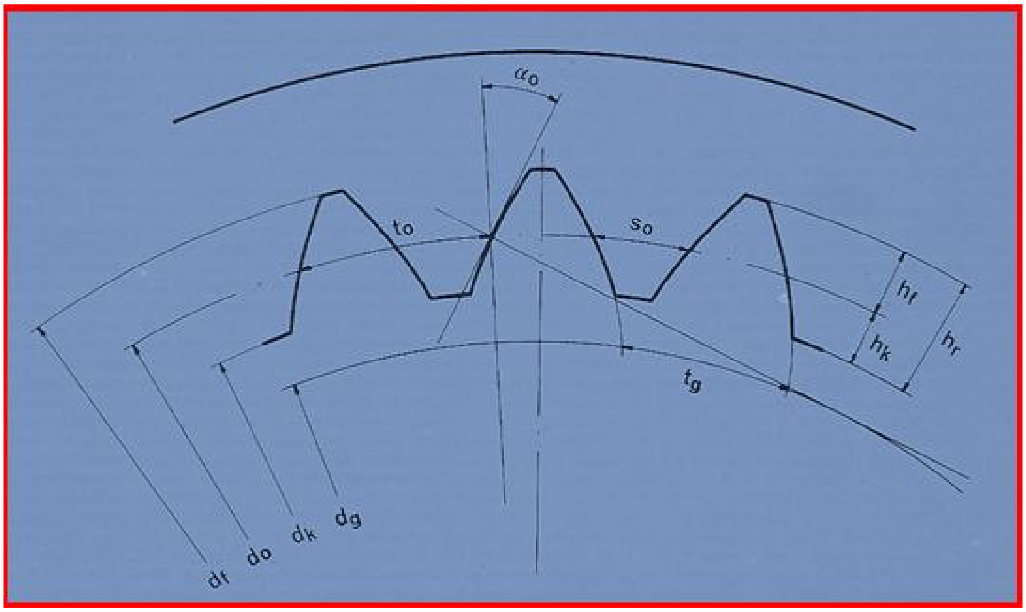

Fig.N°1

Internal spur gears with corrected profile

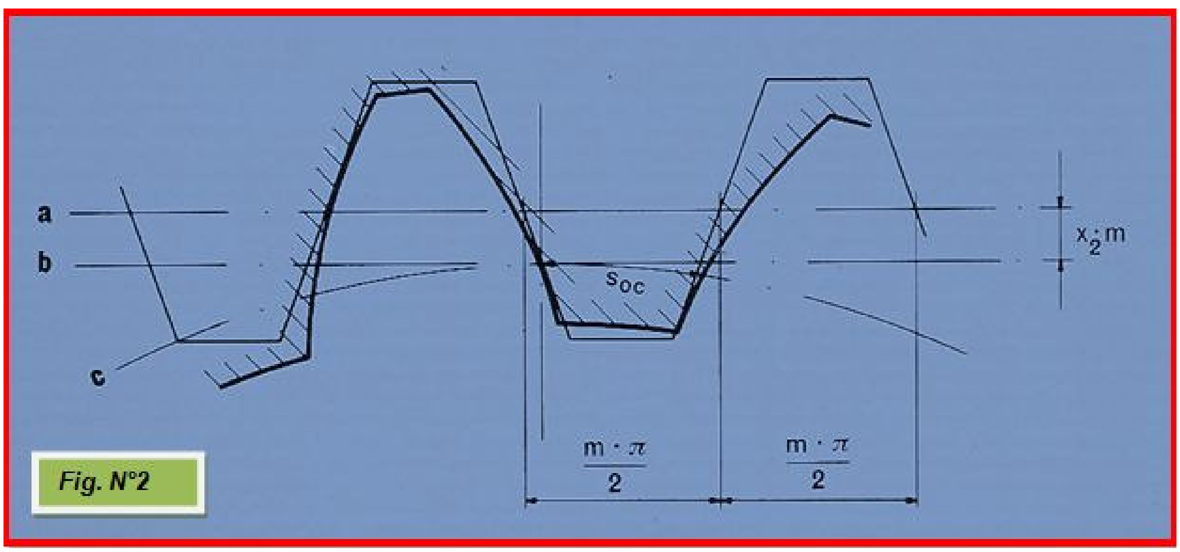

Fig. N°2

a)- Without center distance variation

$$h_k = m - x \cdot m$$

$$h_f = \frac{7}{6} \cdot m + x \cdot m \quad \text{or} \quad h_f = \frac{5}{4} \cdot m + x \cdot m$$

$$s_{oc} = \frac{m \cdot \pi}{2} - 2 \cdot x \cdot m \cdot \tan \alpha_o$$

b)- with center distance variation

$$inv\ \alpha_b = inv\ \alpha_o + 2 \tan \alpha_o \frac{x_1 - x_2}{z_1 - z_2}$$

$$a_b = a \frac{\cos \alpha_o}{\cos \alpha_b} \qquad d_b = \frac{d_g}{\cos \alpha_b}$$

$$s_b = r_b \left[\frac{s_{oc}}{r_o} - 2(inv\ \alpha_o - inv\ \alpha_b)\right]$$
$$d_k = m(z - 2 + 2x)$$

Internal helical gears with normal profile

$$d_o = m_s \cdot z \qquad t_{os} = m_s \cdot \pi$$
$$d_g = d_o \cdot \cos\alpha_{os} \qquad t_{gs} = t_{os} \cdot \cos\alpha_{os}$$
$$m_n = m_s \cdot \cos\beta_o \qquad \tan\alpha_{on} = \tan\alpha_{os} \cdot \cos\beta_o$$
$$t_{on} = m_n \cdot \pi \qquad t_{gn} = t_{on} \cdot \cos\alpha_{on}$$
$$h_k = m_n \qquad h_f = \frac{7}{6} \cdot m_n \quad \text{or} \quad h_f = \frac{5}{4} \cdot m_n$$
$$h_r = h_k + h_f \qquad d_k = d_o - 2h_k$$
$$d_f = d_o + 2 \cdot h_f \qquad s_{on} = \frac{\pi \cdot m_n}{2} \qquad s_{os} = \frac{\pi \cdot m_s}{2}$$

Internal helical gears with corrected profile

a)- Without center distance variation

$$h_k = m_n - x \cdot m_n$$
$$h_f = \frac{7}{6} \cdot m_n + x \cdot m_n \quad \text{or} \quad h_f = \frac{5}{4} \cdot m_n + x \cdot m_n$$
$$s_{onc} = \frac{m_n \cdot \pi}{2} - 2 \cdot x \cdot m_n \cdot \tan\alpha_{on}$$
$$s_{osc} = \frac{m_s \cdot \pi}{2} - 2 \cdot x \cdot m_n \cdot \tan\alpha_{os}$$

b)- With center distance variation

$$inv\ \alpha_{bs} = inv\ \alpha_{os} + 2\tan\alpha_{on} \frac{x_1 - x_2}{z_1 - z_2}$$
$$a_b = a \frac{\cos\alpha_{os}}{\cos\alpha_{bs}} \qquad d_b = \frac{d_g}{\cos\alpha_{bs}}$$
$$s_{bs} = r_b \left[\frac{s_{osc}}{r_o} - 2(inv\ \alpha_{os} - inv\ \alpha_{bs})\right]$$
$$d_k = m_n \left(\frac{z}{\cos\beta_o} - 2 + 2x\right)$$

Contact length calculation

$$\rho_{k1} = \sqrt{r_{k1}^2 - r_{g1}^2} \qquad \rho_{k2} = \sqrt{r_{k2}^2 - r_{g2}^2}$$
$$g = \rho_{k1} - \rho_{k1} + a_b \cdot \sin\alpha_b$$
$$g_1 = \rho_{k1} - r_{b1} \cdot \sin\alpha_b \qquad g_2 = -\rho_{k2} + r_{b2} \cdot \sin\alpha_b$$

Contact radius R_a calculation

$$R_{a2} = \sqrt{(\rho_{k2} + g)^2 + r_{g2}^2}$$

In the case of helical gears use transverse section values α_{bs} instead of α_b .

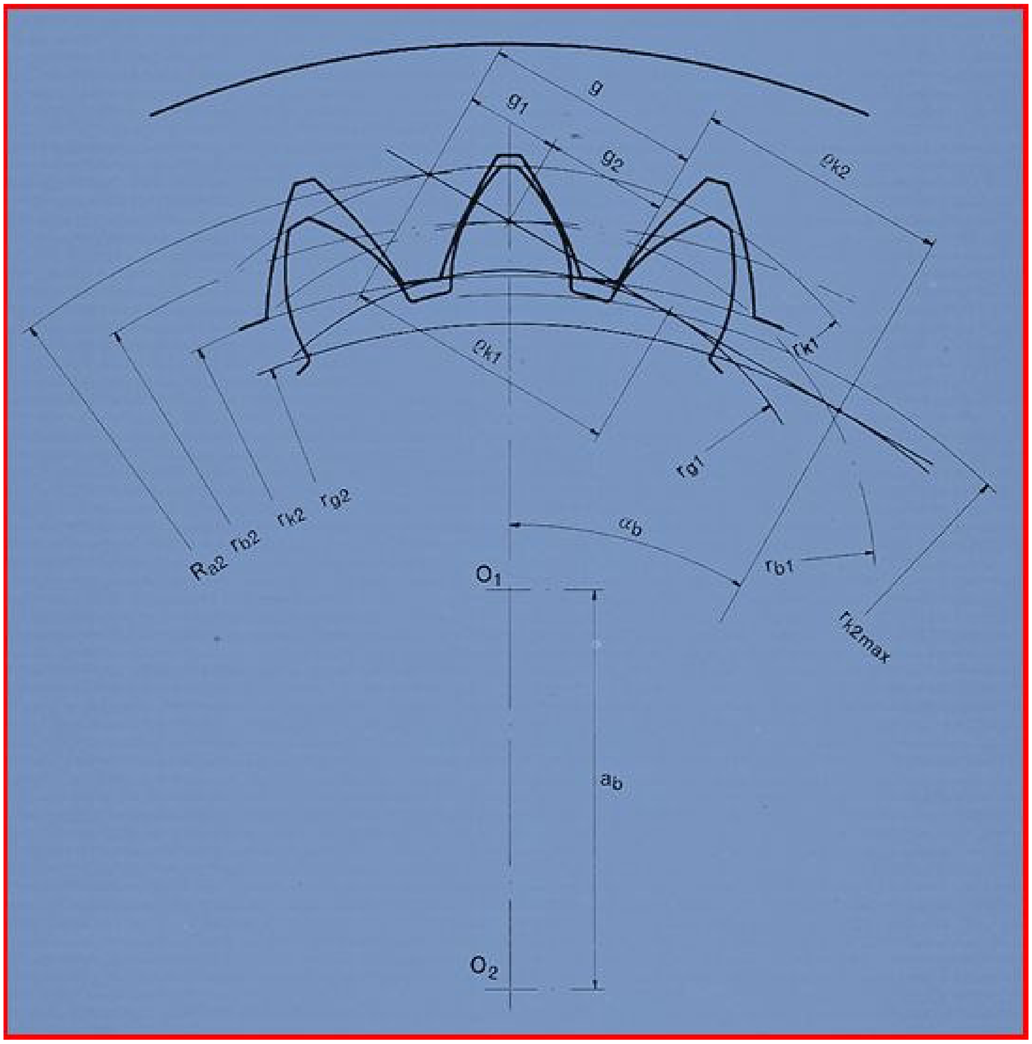

Fig. N°3

Interference

Primary interference

Minimum internal diameter without interference

$$\boldsymbol{d_{k2min} = \sqrt{d_g^2 + (2a_b \cdot \sin \alpha_b)^2}}$$

Secondary interference (figure N°4)

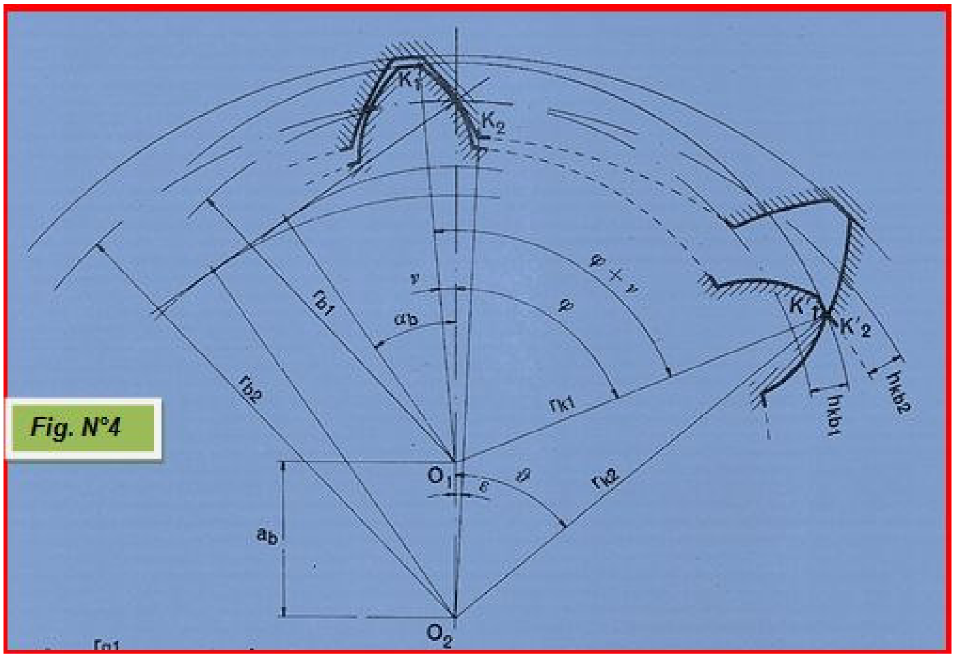

Fig. N°4

$\cos\delta = \frac{r_{g1}}{r_{k1}}$ $\qquad \nu = inv\,\delta - inv\,\alpha_b$

$\cos\partial = \frac{r_{k2}^2 + a_b^2 - r_{k1}^2}{2\cdot r_{k2}\cdot a_b}$ $\qquad \cos\varphi = \frac{r_{k2}^2 - a_b^2 - r_{k1}^2}{2\cdot r_{k1}\cdot a_b}$

When the points K_1 and K_2 on the pinion and gear move to K'_1 and K'_2 in time t_1 and t_2, the respective angles are:

for the gear: $\partial - \varepsilon \;;\quad t_2 = \frac{\partial - \varepsilon}{\omega_2}$

for the pinion: $\varphi + \nu \;;\quad t_1 = \frac{\varphi + \nu}{\omega_1}$

To avoid interference, the points K_1 and K_2 should not coincide at K_1' and K_2' and should satisfy the condition:

$t_1 > t_2$ or $\frac{\varphi + \nu}{\omega_1} > \frac{\partial - \varepsilon}{\omega_2}$

The diagram of figure N°5 is used to determine the largest difference $z_2 - z_1$, which is a function of the pressure angle α_o and the ratio $\frac{h_k}{m}$, where not interference exist.
When the gears are corrected h_k becomes:

$$h_k = \frac{h_{kb2} + h_{kb1}}{2}$$

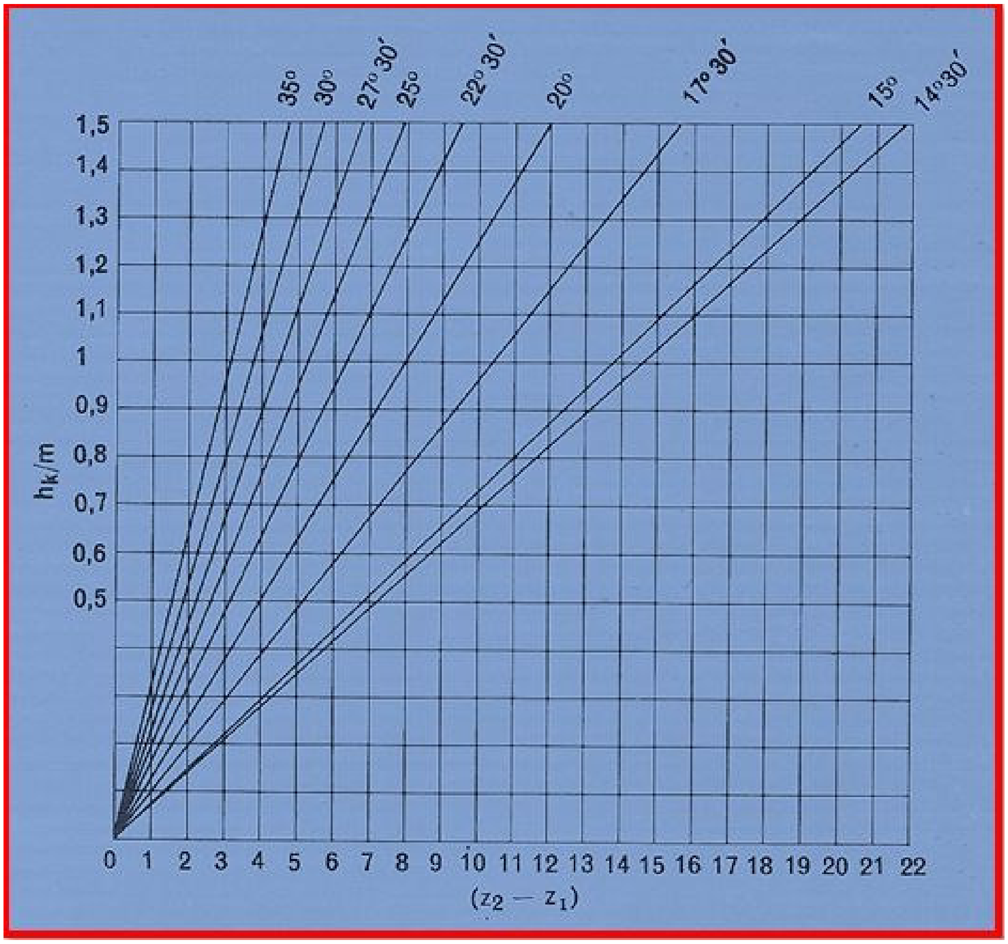

Fig. N°5

Dimension over pins and balls (figure N°6)

Spur gear with even number of teeth

$inv\ \alpha_q = inv\ \alpha_o - \frac{d_r}{2r_o \cdot \cos\alpha_o} + \frac{l_o}{2r_o}$ from which we have α_q

$r_q = r_o \frac{\cos\alpha_o}{\cos\alpha_q}$ $\quad Q = \mathbf{2 \cdot r_q - d_r}$

Spur gear with odd number of teeth

α_q and r_q are the same as for even teeth, but $Q = \mathbf{2 \cdot r_q \cdot \cos\frac{\pi}{2z} - d_r}$

Helical gear with even number of teeth

$$inv\ \alpha_{qs} = inv\ \alpha_{os} - \frac{d_r}{2r_{os} \cdot \cos\beta_o \cos\alpha_{on}} + \frac{l_{os}}{2r_{os}}$$

$r_{qs} = r_{os} \frac{\cos\alpha_{os}}{\cos\alpha_{qs}}$ $\quad Q = \mathbf{2 \cdot r_{qs} - d_r}$

Helical gear with odd number of teeth

α_{qs} and r_{qs} are the same as for even teeth, but $Q = \mathbf{2 \cdot r_{qs} \cdot \cos\frac{\pi}{2z} - d_r}$

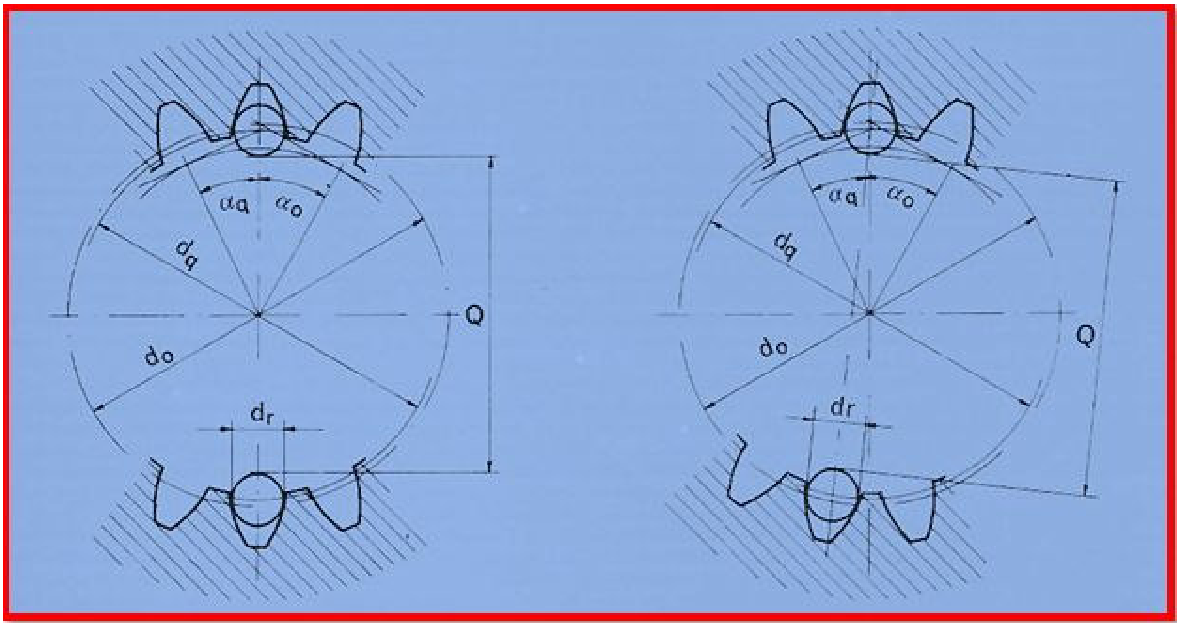

Fig.N°6