Considerations on workpiece errors

It has already been stated that by increasing the number of gashes the accuracy of the profile improves.
To tell the truth, however, theoretical profile errors caused by enveloping are few and they are well outnumbered by other errors that are generated by other causes.
Lead errors that are generated by feed scallop marks are more important and these depend both on the value of feed per workpiece revolution and on the diameter of the hob.
These scallop are very common in gears and workshop operators can often estimate off hand the feed per workpiece revolution just by observing the pitch of these groove marks.
This error may be very limiting especially if the gear is to be finished b shaving.
It is in fact well known that many more errors are tolerated if a gear is to be finished by grinding operation.
With reference to figure N°1

$$\varepsilon_1 = r \cdot tg\alpha \cdot \left(1 - \cos\frac{\eta}{2}\right) \qquad \text{where} \qquad \eta = \frac{360 \cdot Z_0}{Z \cdot i_0}$$

$$\varepsilon_2 = \frac{f_a^2 \cdot \cos^2\beta \cdot tg\alpha}{8 \cdot r}$$

The note used are the following:

Z_0 = number of start of the hob
i_0 = number of gashes of the hob
Z = number of gear teeth
r = Pitch radius of hob
f_a = feed per workpiece revolution
β = thread angle of hob
α = pressure angle

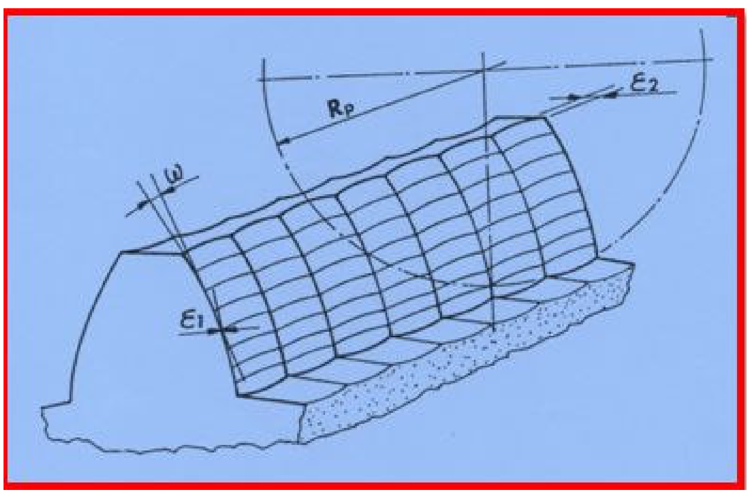

Fig. N°1

As an example let us consider the following case:

- Gear: m = 2 ; Z = 47 ; α = 20° ; β = 15°
- Hob : Outside diameter 100 mm (pitch diameter around 95 mm); 3 starts ;

- for $i_0 = 15$ $\varepsilon_p = 1{,}54\mu$
- for $i_0 = 22$ $\varepsilon_p = 0{,}71\mu$
- for $i_0 = 28$ $\varepsilon_p = 0{,}44\mu$
- for $f_a = 3{,}0$ mm/rev $\varepsilon_x = 8{,}5\mu$
- for $f_a = 5{,}6$ mm/rev $\varepsilon_x = 29{,}6\mu$

If the outside diameter of the hob is 50 mm (pitch diameter about 45mm), the following applies:

- for $f_a = 4{,}4$ mm/rev $\varepsilon_x = 36{,}5\mu$

As can be seen there may be very significant lead errors which cannot be neglected.
It is also necessary to consider that these are the theoretical errors.
Basically we must add other errors to these which may be generated by numerous causes like for example: run out errors in mounting the hob, errors in the construction of the hob itself, resharpening errors, machine vibration, scratches and so on.
Some considerations regarding the distribution of the feed marks left on the gear b the hob may be made with regard to the classical figure N°1.
In fact the entity may be calculated with the above formulas but it's interesting to note the difference between the marks left on the gear by a single-start hob and a hob with two starts which have the same number of gashes and which cut a gear in the same amount of time.
In the figure N°2 the width of the axial feed marks is clearly shown.
There are less than half because the feed per workpiece revolution is halved while the radial marks are doubled (envelope) because the number of gashes involved in the formation of the tooth is half.
This however does not means that the profile error ε_1 doubles as can be seen in the formula used to calculate it.

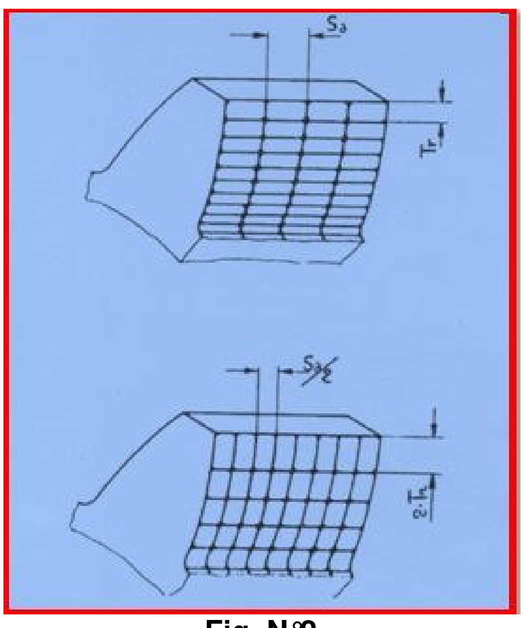

Fig. N°2

The number of gashes is also important in terms of the distribution of grooving. When using hobs with a single start, the feed marks are aligned as shown in figure N°1 whereas with multi-start hobs, the feed marks are only aligned when the number of starts divides the number of gashes exactly. If the number of starts and the number of gashes have a prime relation or if they are in an case divisible, the distribution of the feed marks is “staggered” as a figure N°3.

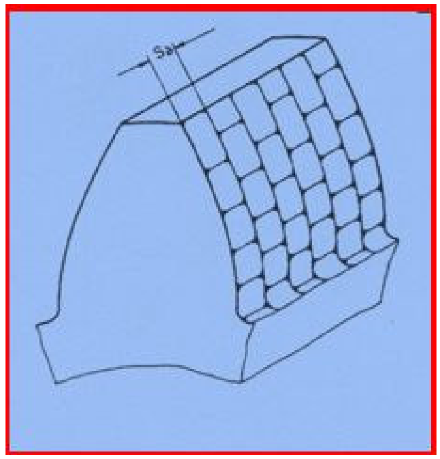

Fig. N°3

Errors in manufacturing the hob and in mounting it on the machine are very important in terms of accuracy that is attainable on in the workpiece.
In particular hob run out during manufacturing directly influences the quality of the profile. Hob run out may be due to errors in construction of the hob itself or errors in resharpening but, more often than not, they are generated by the hob being incorrectly mounted on the machine.
There are three different types of run out error:

a) *Uniform run out on the whole hob. This is when the hob axis and the axis of rotation are parallel but they do not coincide.*
b) *Run out at the two extremities of the hob. This occurs when the two axes (of the hob and of rotation) are not parallel and they intersect half way up the hob.*
c) *Run out at just one of the extremities of the hob. This is generated by two non-parallel axes which intersect at an extremity of the hob*

In case (a) figure N°4, the rack which is made up of a row of teeth reciprocates to and from its theoretical position in a sinusoidal manner, keeping the tooth tips parallel to the hob axis so that the tooth which generated has smaller or larger cordal thickness in its different sections to theoretical measurements. Basically the profile has a sinusoidal shape and the two sinusoids are offset by 180° as shown in the figure N°4.

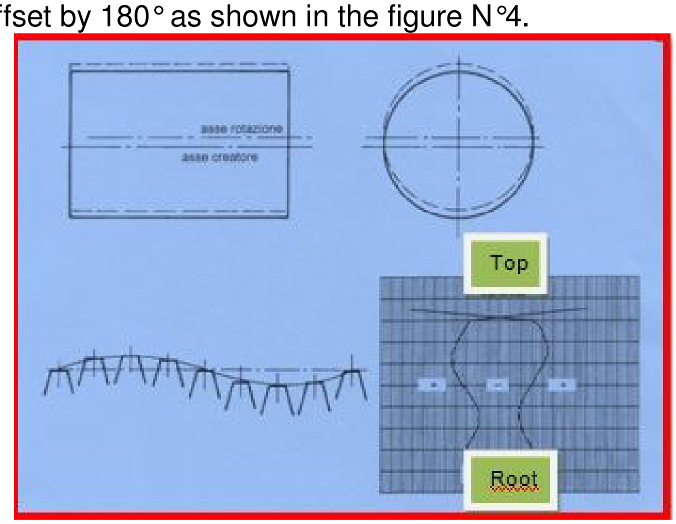

Fig. N°4

In cases (b) and (c) the teeth reciprocate to and from the theoretical position but the inclination of the hob teeth also continuously varies since their axis follows the sinusoidal form.
If the pressure angle is increased on one flank, it decreases on the opposite flank.
Basically whit the sinusoidal form, a profile error is generated but there are two sinusoids (one for each of the two flanks) phased together (see figure N°5).
This means that the tooth thickness is correct in all sections but it is as if the tooth where distorted.
Naturally the entity of these errors in absolute values depends on the run out value.

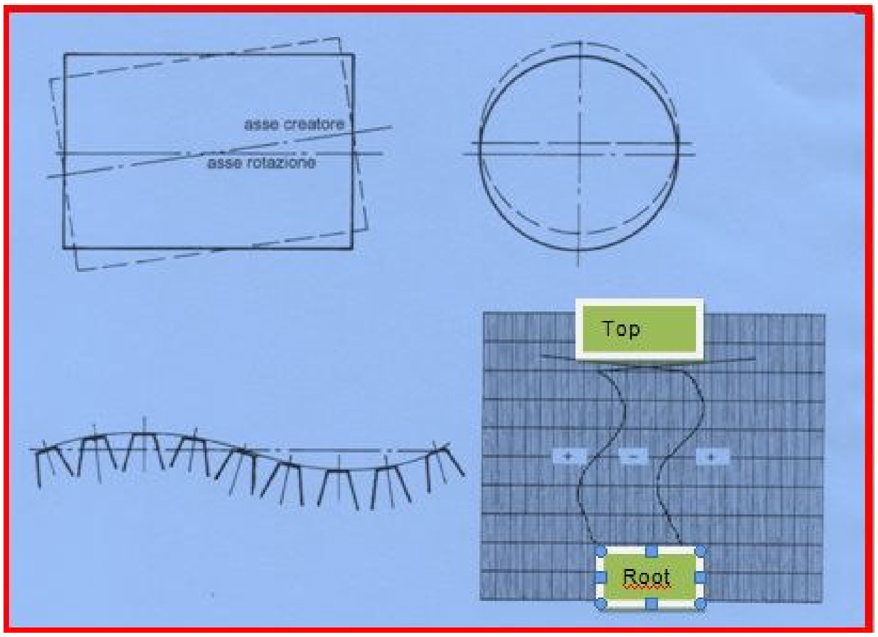

Fig. N°5